Conjugate natural convection of a power law fluid in a vertical finite thick channel with heat sources

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Nomenclature

T_1, T	=	Temperatures of walls and fluid
k_1, k	=	Thermal conductives of walls and fluid
T_w	=	Temperature on external sides of walls
T_∞	=	Temperature of surroundings
ρ	=	Fluid density
v_k	=	Kinematic consistency
C_p	=	Specific heat
δ	=	Thickness of walls
Ω, Ω_*	=	Continuous and grid regions
~	=	Denotes dimensionless quantities
H	=	The height of the channel
P_{rg}	=	Generalized Prandtl number
Table I	=	Input parameters
Figures 1-8a	=	Velocity fields corresponding to the cases 1-8
Figures 1-8c	=	Surfaces of heat distribution of the cases 1-8
t	=	Time
x, y	=	Co-ordinates, x along channel axis
u, v	=	Velocities in the x and y directions
u_0	=	Velocity at channel entry
P'	=	Pressure imbalance
β	=	Thermal expansion coefficient
Q	=	Average heat transfer parameter
Q_1, Q_2	=	Uniform heat sources
b	=	Interplate spacing
S_1, S_2	=	Dimensionless heat sources
n	=	Index of power law fluid
G_{rg}	=	Generalized Grashof number
Table II	=	Characteristics parameters from the results of computation
Figures 1-8b	=	Isothermal lines corresponding to the cases 1-8

Introduction

The study of heat transfer in the natural convection of non-Newtonian fluid in a channel or pipe is one of the most basic and important engineering mechanics problems. In particular, the model of power law fluid has become a focus of interest because of its suitability and applicability in practice. Many works have been published in this field, such as Irvine *et al.*[1], Acrivos[2], Tien[3], Shenoy

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and Mashelkar[4], Cho and Hartnett[5] and Moses and Hartnett[6], in which the natural convection of non-Newtonian fluid has been studied.

In this paper, the extension is made by introducing different heat sources in both the fluid and channel walls. At the same time, the influence of the heat transfer into the channel walls is also taken into account. Omission of the effect of wall thickness can cause inaccuracy in numerical simulation of problems in practice.

Fourier's equation in the general case and in Cartesian co-ordinates takes the following form,

$$\frac{\partial T_1}{\partial t}=\alpha\left(\frac{\partial^2 T_1}{\partial x^2}+\frac{\partial^2 T_1}{\partial y^2}+\frac{\partial^2 T_1}{\partial z^2}\right)+\frac{1}{\rho c}f(x,y,z,t) \tag{1}$$

in which T_1 is the temperature inside the walls, α the thermal diffusivity, t the time and $f(x, y, z, t)$ the heat source placed in the walls. However, here the paper assumes that the process of heat-transfer is steady, one dimensional and includes uniform heat sources in the walls as well as the fluid. This situation, perhaps, is suitable in the case when the plate thickness is large enough not to be ignored but small enough in comparison with the two other dimensions. The equation therefore can be reduced to the one dimensional equation.

The natural convection equation with boundary conditions is solved using a finite difference method. The fields of velocity and temperature, as well as dimensionless characteristics of heat exchange and the averaged velocity, are determined.

The governing equations and boundary conditions

For the steady 2D convection heat transfer with a power law fluid in a vertical channel of thick walls and with heat sources distributed uniformly in both the fluid and the walls, and assuming that the heat transfer into the walls only occurs in a horizontal direction owing to the channel thickness being very small in comparison with the channel height, the governing equations are as follows,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp'}{dx}+\nu_k\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n+g\beta(T-T_\infty) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho C_p}Q_2, \qquad -\frac{b}{2}\le y\le\frac{b}{2},\ 0\le x\le H \tag{4}$$

$$k_1\frac{d^2T_1}{dy^2}=-Q_1, \quad -\frac{b}{2}-\delta\le y\le-\frac{b}{2} \quad or \quad \frac{b}{2}\le y\le\frac{b}{2}+\delta,\ 0\le x\le H \tag{5}$$

in which u and v are the velocities in the x and y directions respectively, with the x axis along the channel; ρ the fluid density; p' the pressure imbalance; v_k the kinematic consistency; n the flow behaviour index for power law fluid; g the acceleration of gravity; β the thermal expansion coefficient; T the temperature of fluid; T_∞ the temperature of the surroundings; k the thermal conductivity; C_p the specific heat; b the interplate spacing; δ the thickness of the walls; Q_1 and Q_2 the uniform heat sources placed in the walls and in the fluid respectively; and T_w the temperature on the external sides of the walls. The pressure imbalance, p', is defined by

$$p'(x)=p(x)-p_0+\rho_\infty gx \tag{6}$$

where p_0 and ρ_∞ are outside ambient pressure and density respectively. Introducing the following dimensionless variables,

$$\tilde{x}=\frac{x}{H},\ \tilde{y}=\frac{y}{b},\ \tilde{u}=\frac{bu}{Hu^*},\ \tilde{v}=\frac{v}{u^*},\ \tilde{\delta}=\frac{\delta}{b}$$

$$\tilde{T}=\frac{T-T_\infty}{T_w-T_\infty},\ \tilde{T}_1=\frac{T_1-T_\infty}{T_w-T_\infty},\ \tilde{p}'=\frac{p'b^2}{\rho u^{*2}H^2}$$

$$P_{rg}=\frac{\rho C_p}{k}u^*b,\ G_{rg}=\frac{g\beta(T_w-T_\infty)b^2}{Hu^{*2}},\ u^*=\frac{v_k^{\frac{1}{2-n}}b^{\frac{1-2n}{2-n}}}{H^{\frac{1-n}{2-n}}}$$

$$S_1=\frac{Q_1b^2}{k_1(T_w-T_\infty)},\ S_2=\frac{bQ_2}{\rho C_p u^*(T_w-T_\infty)} \tag{7}$$

and dropping tilde signs for convenience, we obtain the basic equations in dimensionless form as follows,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{dp'}{dx}+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n+G_{rg}T \tag{9}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{P_{rg}}\frac{\partial^2 T}{\partial y^2}+S_2, \quad -\frac{1}{2}\le y\le\frac{1}{2}, \quad 0\le x\le 1 \tag{10}$$

$$\frac{d^2T_1}{dy^2}=-S_1, \qquad -\frac{1}{2}-\delta\le y\le -\frac{1}{2} \;\; or \;\; \frac{1}{2}\le y\le \frac{1}{2}+\delta,\; 0\le x\le 1 \tag{11}$$

Because of the symmetry, we only consider a half-channel flow field. In this case, the boundary conditions are as follows,

$$\begin{aligned} &u(0,y)=u_0, \quad v(0,y)=0,\\ &T(0,y)=0, \quad p'(0,y)=0, \end{aligned} \tag{12}$$

$$\begin{gathered} u(x,\frac{1}{2})=0, \quad v(x,\frac{1}{2})=0\\ \frac{\partial u}{\partial y}(x,0)=0, \quad v(x,0)=0\\ \frac{\partial T}{\partial y}(x,0)=0, \quad p'(1,y)=0,\\ T(x,y)\big|_{y=\frac{1}{2}}=T_1\big|_{y=\frac{1}{2}} \end{gathered}$$

$$T_1(y=\frac{1}{2}+\delta)=1, \quad k\frac{dT}{dy}\big|_{y=\frac{1}{2}}=k_1\frac{dT_1}{dy}\big|_{y=\frac{1}{2}}$$

where k_1 is the thermal conductivity for walls. It should be noted that the three last boundary conditions are introduced to close the problem and the last one is considered as the conjugate condition.

Numerical solutions and discussions

Let Ω be the continuous region of consideration for the fluid

$$\Omega=\{(x,y):0\le x\le 1,\; 0\le y\le 1/2\}$$

and Ω_* be the corresponding grid region

$$\Omega_*=\{(i,j):1\le i\le N,\; 1\le j\le J\}$$

in which Δy is constant and Δx is variable. Using the three-point difference formulae, equations (8)-(11) and the boundary conditions (12) can be easily discretized. Owing to the feature of difference formulae, the equations (10) and (11) are solved first. The difference equation corresponding to equation (10) is,

$$a_iT_{i-1}^{j+1}+b_iT_i^{j+1}+c_iT_{i+1}^{j+1}=d_i, \quad (i=2,N-1) \tag{13}$$

in which,

$$a_i=\frac{\lambda}{P_{rg}}+\tau v_i^j,\quad b_i=-\frac{2\lambda}{P_{rg}}-u_i^j,$$
$$c_i=\frac{\lambda}{P_{rg}}-\tau v_i^j,\quad d_i=-u_i^jT_i^j-\Delta x S_2,$$
$$\tau=\frac{\Delta x}{2\Delta y},\quad \lambda=\frac{\Delta x}{\Delta y^2},\quad (i=2,N-1)$$

In order to get the boundary condition for equation (13) at y = 1/2 to correspond with i = N, it is necessary to integrate equation (11) and then to use the above conjugate condition, with the results,

$$T_N^{j+1}(1+\frac{k_1}{k}\frac{\Delta y}{\delta})-T_{N-1}^{j+1}=\frac{k_1\Delta y}{k\delta}\left[-\frac{S_1\delta}{2}+\frac{S_1}{2}\left(\frac{1}{2}+\delta\right)^2-\frac{S_1}{8}+1\right] \tag{14}$$

Applying the finite difference method to the boundary condition at y = 0 for T, we obtain

$$T_1^{j+1}-T_2^{j+1}=0 \tag{15}$$

It is clear that the equations (13)-(15) represent a tridiagonal linear equation system which is easy to solve. Next the difference equations corresponding to equation (9) are solved for u and p'. However, because of the appearance of p' in equation (9), a supplement equation is required. In a way similar to that of Irvine *et al.*[1], we introduce the condition representing the conservation of fluid mass in the channel:

$$\int_0^{\frac{1}{2}}udy=\frac{1}{2}u_0 \tag{16}$$

For equation (8), we can directly integrate on the base of the given solution of u. Thus with initial estimate[1] of u_0, the iteration is done until p' = 0 at x = 1.

In order to illustrate the influences of the wall thickness as well as the heat sources in the paper, several concrete cases have been computed as examples. The common input data for all the cases are shown in Table I, except for case 9

Table I.
Input parameters

T_w	=	25°C	T_∞	=	15°C
b	=	2cm	H	=	20cm
ρ	=	1,000kg/m^3	C_p	=	4.18 10^{3}J/kg.K
k	=	0.597W/m.K	v_k	=	7.35 10^{-5}m^2/s^{2-n}
β	=	1.8 10^{-4}K^{-1}	n	=	0.66
δ	=	b/8			

with $\delta = 0$, which has been studied before[1]. However, it should be noted that only the cases where the channel walls of thermal conductivity are four and ten times larger than that of fluid are considered here. The algorithm has been coded in FORTRAN 77 language (FTN77/386) to run under graphics mode, so the computation results are presented by curves, facilitating the understanding of the behaviour of solutions and enabling the easy adjustment of the value of u_0 at the entrance to the appropriate value.

The computational results for cases 1-8 are shown in Table II and in Figures 1-8; case 9 is that of Irvine *et al.*[1]. The final results of interest for different cases are the characteristics parameters: the velocity at channel entry u_0 and the average heat transfer Q (see Table II).

Cases	k_1/k	Q_1(KW/m^3)	Q_2(KW/m^3)	u_0(cm/s)	Q(W/m)
1	10	3	3	1.3	475.03
2	4	3	3	1.22	460.5
3	10	10	10	1.34	475.25
4	10	10	0	1.28	475.06
5	10	50	0	1.29	475.52
6	10	0	50	1.55	475.95
7	4	0	0	1.21	460.23
8	10	0	0	1.27	474.94
9	∞	0	0	1.33	486.0

Table II. The characteristics parameters from the results of computation

Figures 1a-8a present the velocity vector fields in the plane of half channel, Figures 1b-8b present-isothermal lines and Figures 1c-8c are the surfaces of heat distribution for cases 1-8 respectively. A comparison of the results obtained in Table II, as well as in Figures 1-8, gives rise to the following remarks:

- Cases 7-8 in Table II and the results of computations illustrated in Figures 7-8 reflect the influence of heat transfer into the channel wall; the velocity u_0 at entrance and the average heat transfer Q decreased considerably in comparison with case 9. At the same time, it is also shown that this influence will be ignored when $k_1/k >> 1$, i.e. it becomes the case of the channel without thickness (case 9).
- Cases 1-6 present the influence of both the heat sources and the thickness of channel at different levels which are illustrated in Table II and Figures 1-6. The existence of heat sources under 50kW/m^3 does not change the distribution of heat very much in the channel. However, this effect will be large when the heat source is from 50kW/m^3 and placed in the fluid (case 6, Figure 6).
- In all the cases, it can be seen that next to the channel entrance flow is directed towards the channel axis, and the intensity of flow velocities are very large, while elsewhere the flow is nearly parallel to the channel axis.

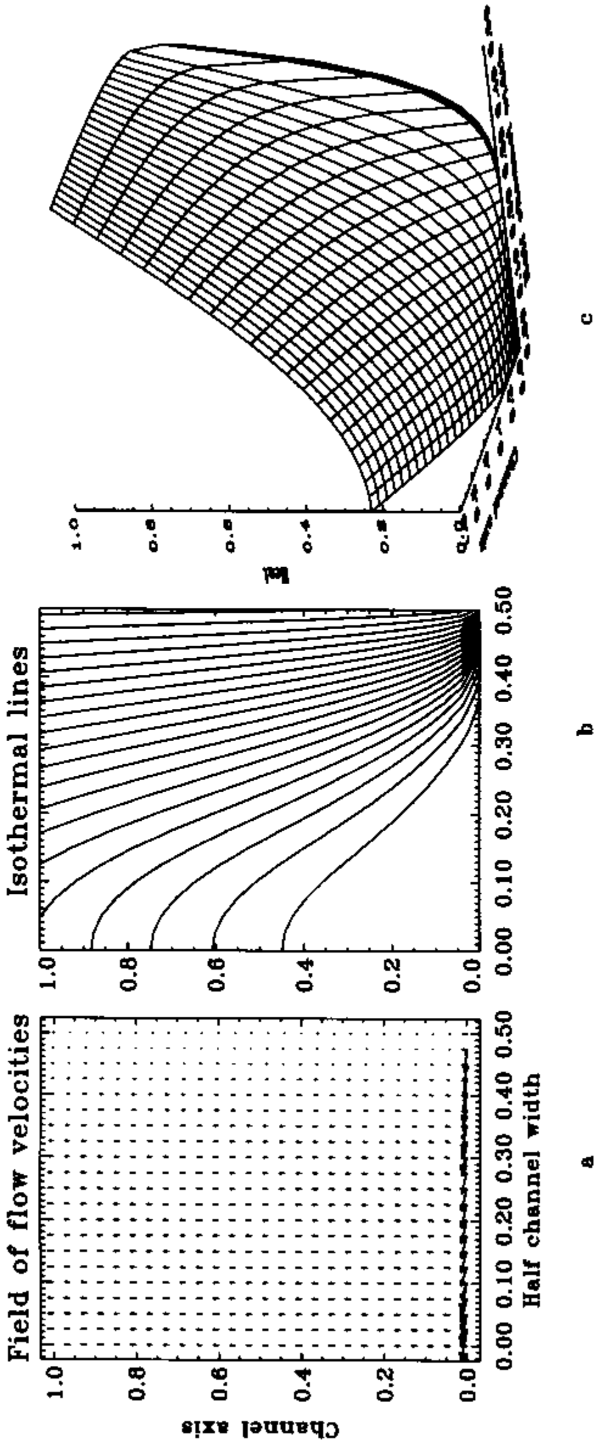

Figure 1.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 1

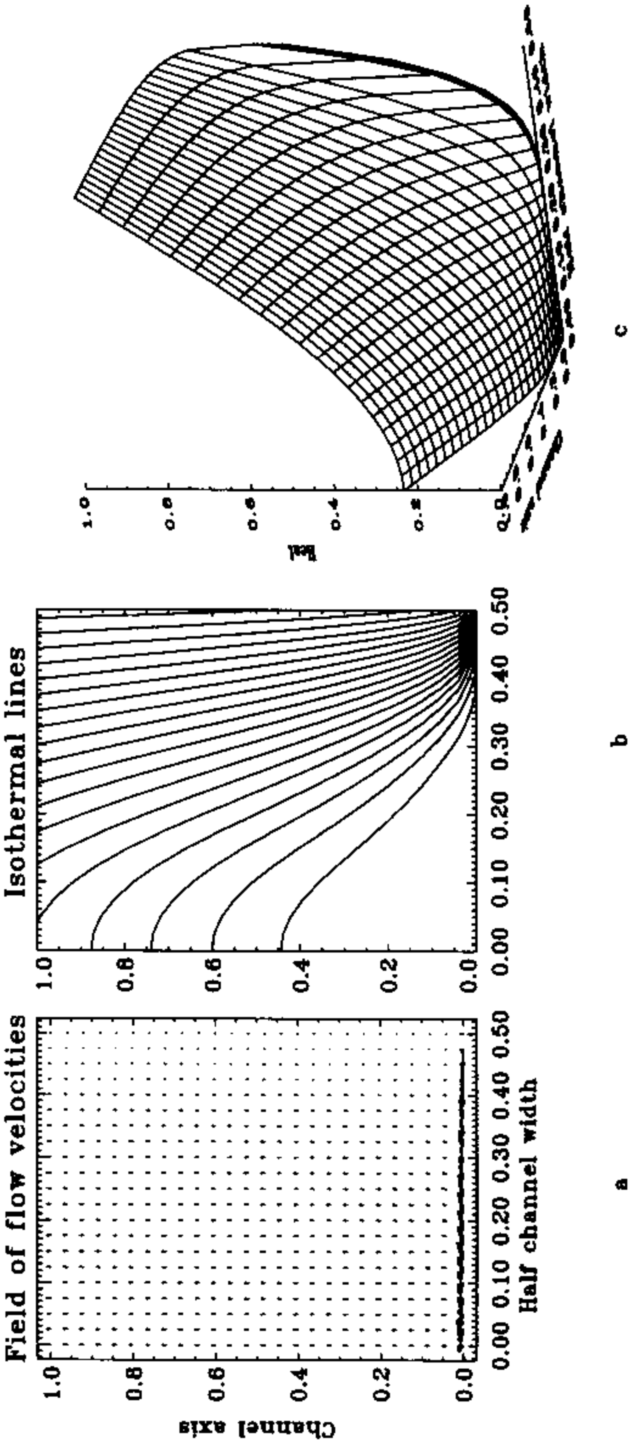

Figure 2.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 2

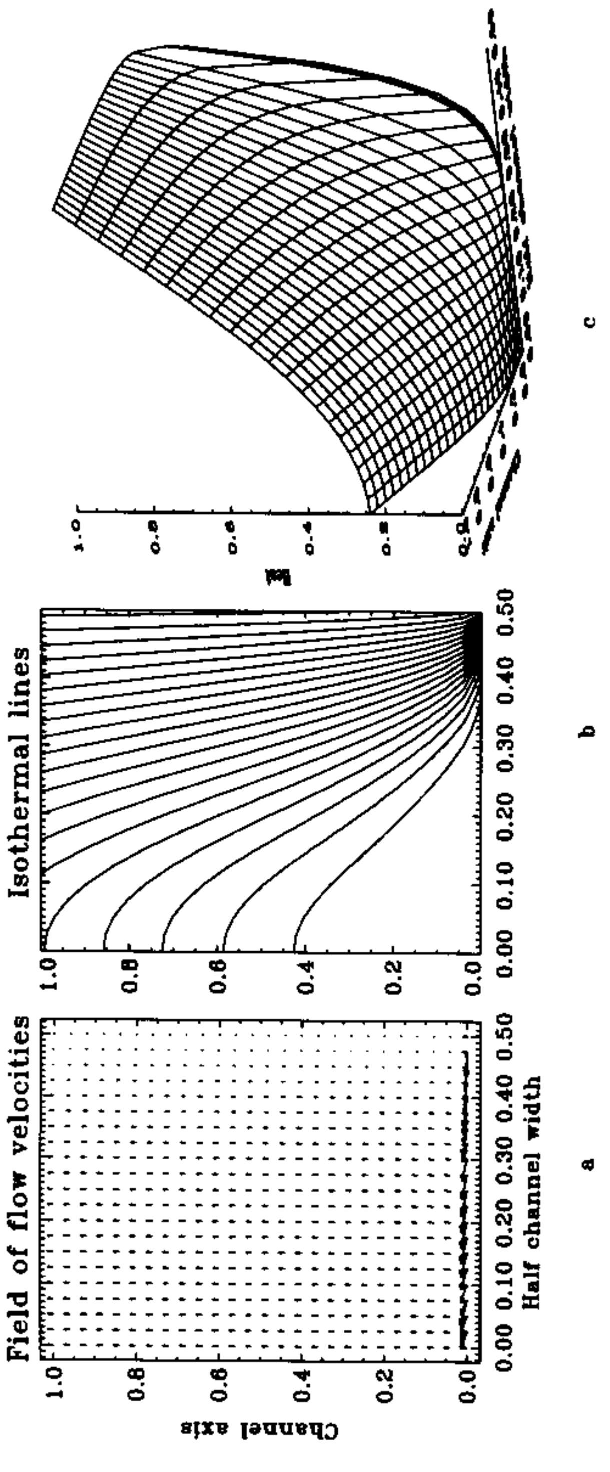

Figure 3.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 3

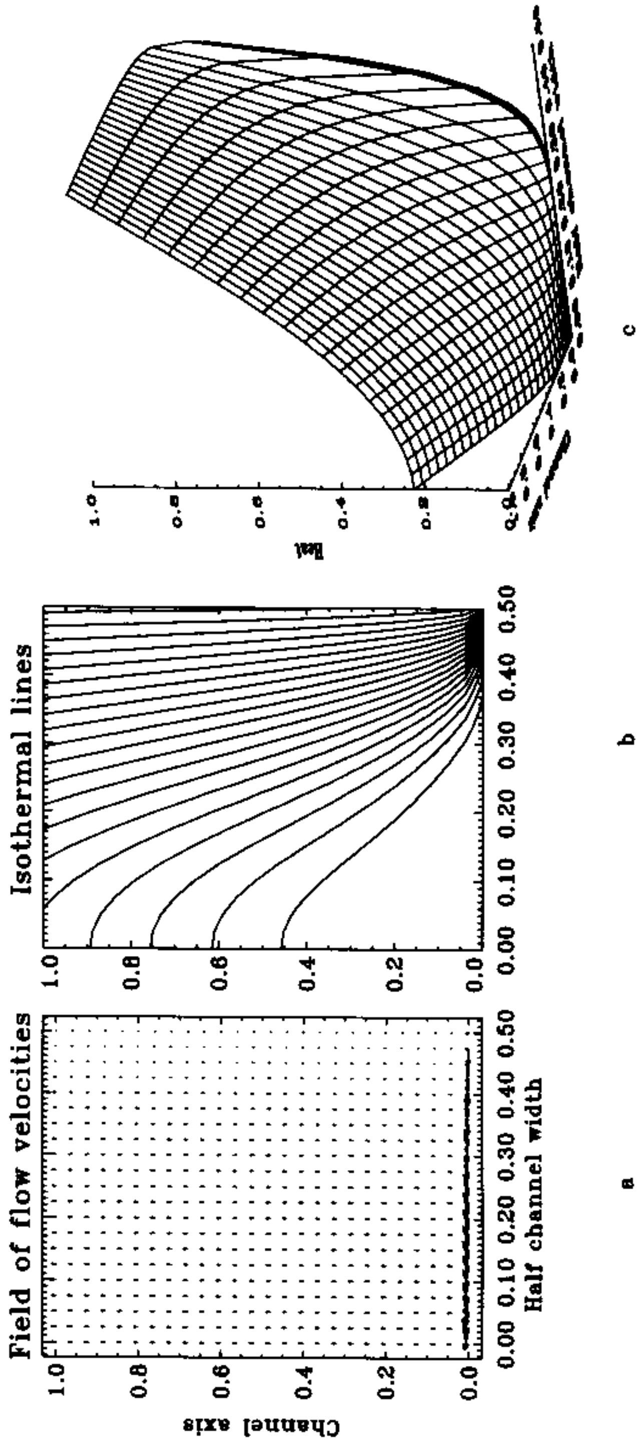

Figure 4.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 4

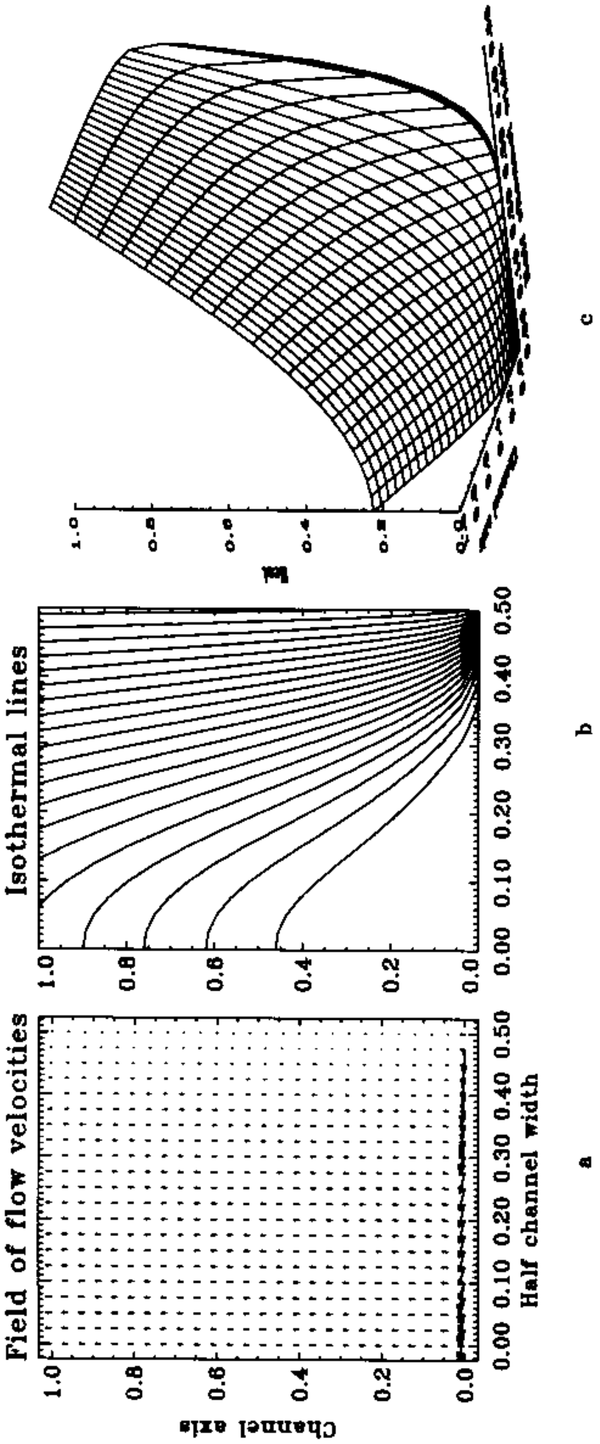

Figure 5.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 5

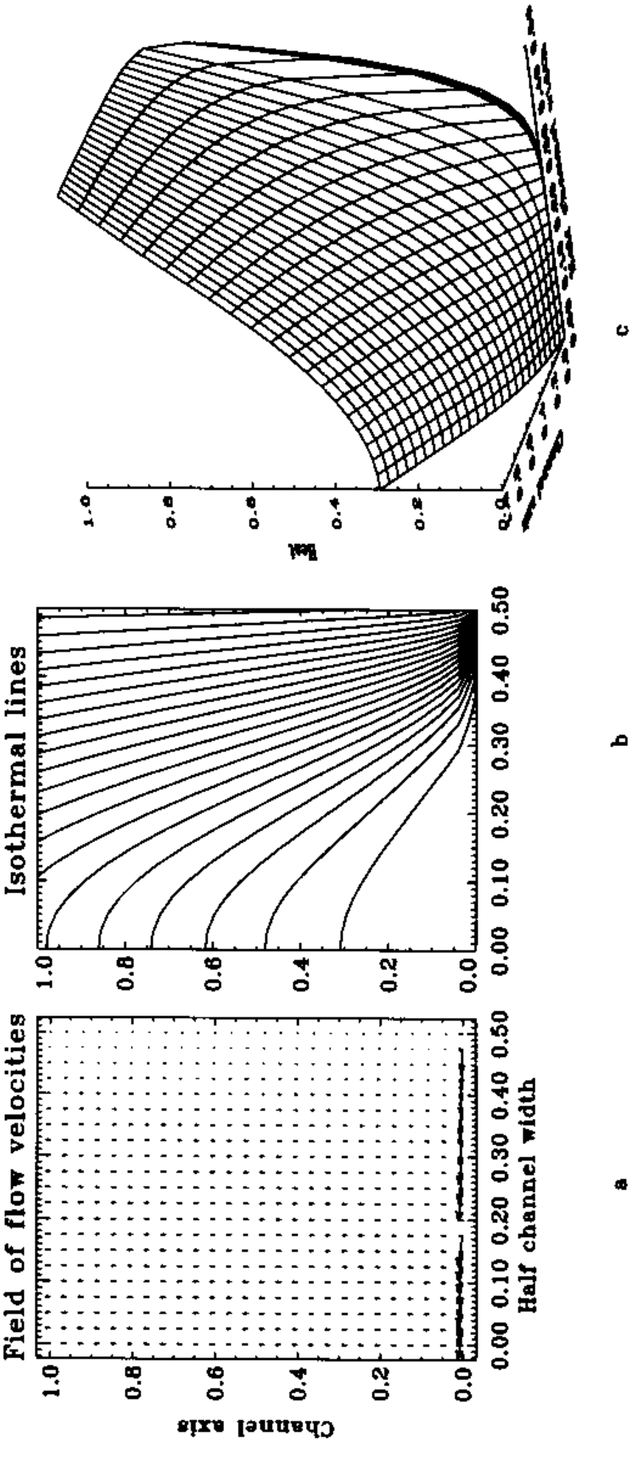

Figure 6.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 6

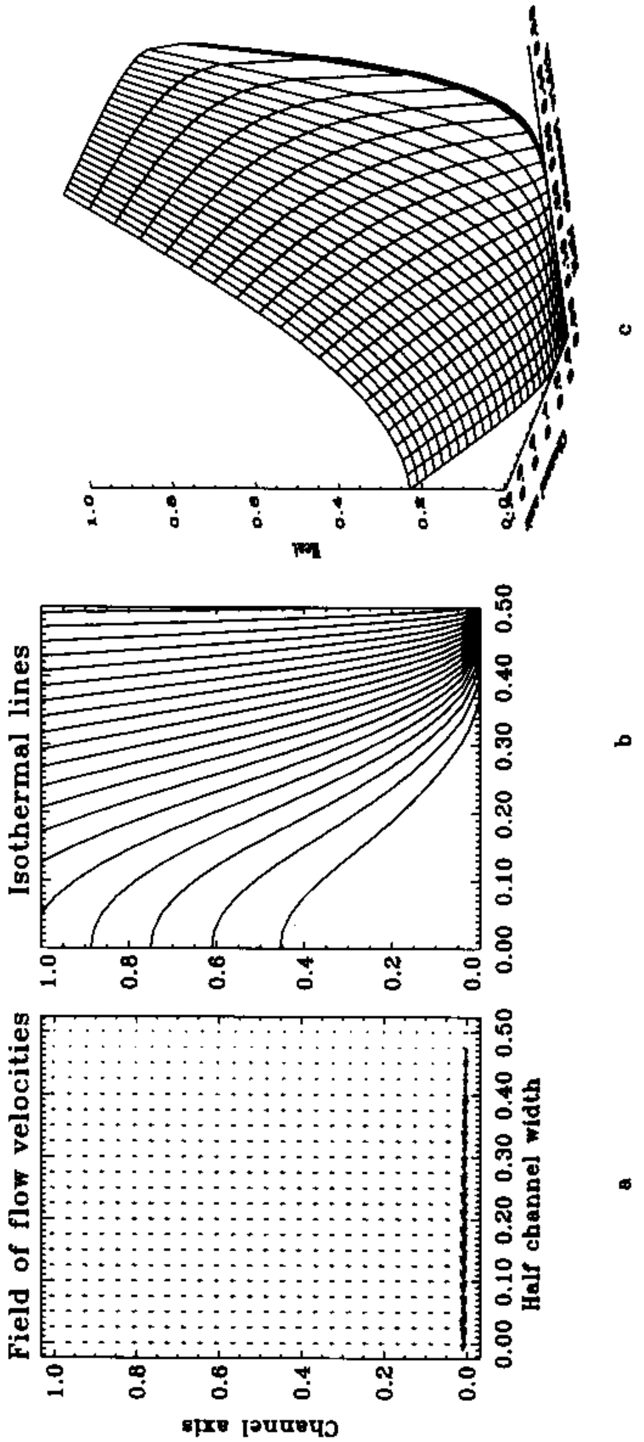

Figure 7.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 7

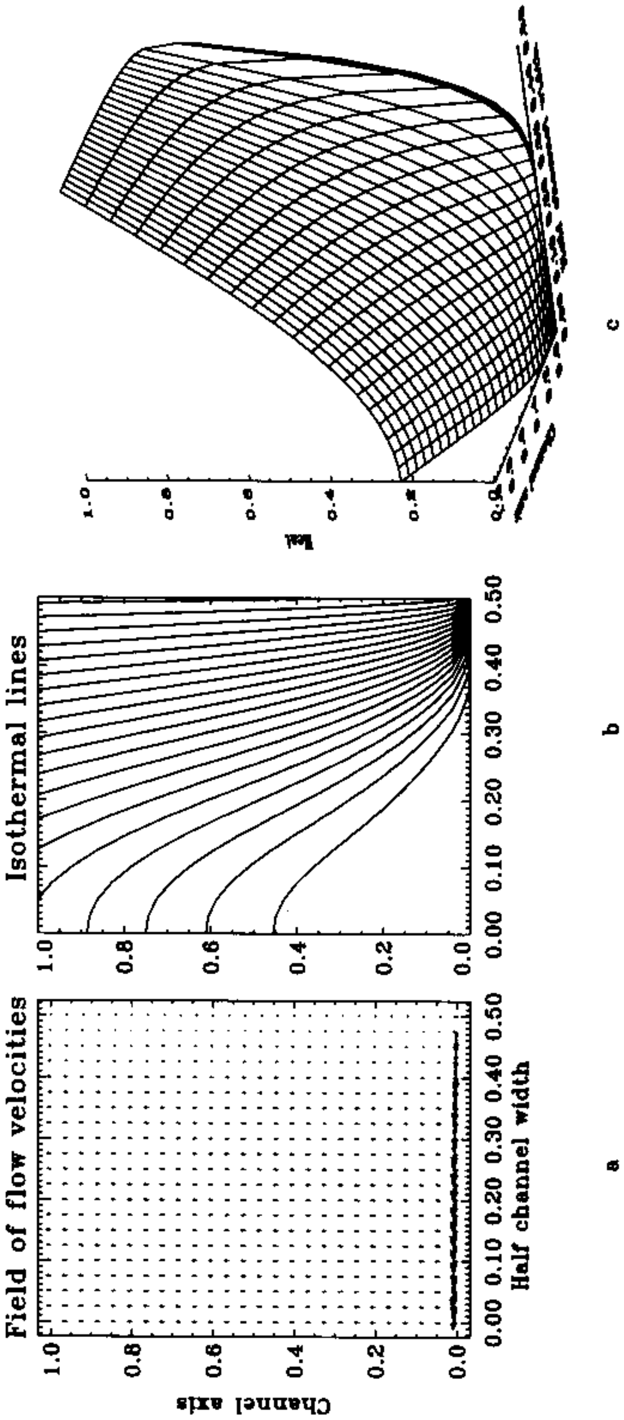

Figure 8.
Map of isothermal lines (a), velocity field (b) and heat distribution in half channel (c) for case 8

Conclusion

Using a finite difference method, the governing equations for the flow of a power law fluid in a finite vertical channel of thick walls with heat sources uniformly distributed have been solved completely. The fields of temperature and fluid velocities as well as average heat transfer, illustrating numerical results, have been obtained for different cases. The results showed that the influences of wall thickness and heat sources will become considerable when the substance channel wall has small thermal conductivity and the heat sources are large enough.

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